

[Total No. of Questions - 9] [Total No. of Printed Pages - 4]

Dec.-23-0364
MA-301 (Probability & Statistics)
(Common for B.Tech. all Branch)
B.Tech. 3rd (CBCS)

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : The candidates are required to attempt at least one question carrying 10 marks from each section A, B, C, D and ten questions carrying 2 marks each from section E.

SECTION - A

1. A witness to a crime scene briefly notices the convict who escaped. While enquired, the witness says the convict person is a female. Testing the witness under similar conditions shows the witness correctly identifies gender of a person 80% of the time. According to the current population survey, 90% people of the city are male and 10% are female. Using Bayes' theorem, find the probability that the escaped person was actually female? (10)

2. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

Determine the value of a, mean, variance and cumulative distribution function. (10)

SECTION - B

3. According to **Chemical Engineering Prog. Journal**, approximately 30% of all pipework failures in chemical plants are caused by operator error that is discretely distributed.

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- (i) What is the probability that out of the random sample of 20 such failures, exactly 5 are due to operator error?
- (ii) What is the probability that no more than 4 out of 20 such failures are due to operator error? (10)

4. A soft drink machine is regulated so that it discharges an average of 200 millilitres per cup. If the amount of drink follows Normal distribution with a variance equal to 225,

- (i) What is probability that a cup contains between 191 to 209 millilitres?
- (ii) How many cups will probably overflow if 230 millilitre cups are used for the next 1000 drinks?
- (iii) Below what value do we get the smallest 25% of the drinks? (10)

SECTION - C

5. The television picture tubes of manufacturer A have a mean lifetime of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean lifetime of 6.0 years and a standard deviation of 0.8 year. What is the probability that a random sample of 36 tubes from manufacturer A will have a mean lifetime that is at least 1 year more than the mean lifetime of a sample of 49 tubes from manufacturer B? (10)

6. A machine produces metal pieces that are cylindrical in shape. A sample of these pieces is taken and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimetres. For all computations, assume an approximately normal distribution. The sample mean and standard deviation for the given data are, $\bar{x} = 1.0056$ and $s = 0.0246$ respectively.

- (i) Find a 99% confidence interval on the mean diameter.

[P.T.O.]

- (ii) Compute a 99% prediction interval on a measured diameter of a single metal piece taken from the machine.
- (iii) Find the 99% tolerance limits that will contain 95% of the metal pieces produced by this machine. (10)

SECTION - D

7. In a study to estimate the proportion of residents in a certain city and its suburbs who favour the construction of a nuclear power plant, it is found that 63 of 100 urban residents favour the construction while only 59 of 125 suburban residents are in favour. Is there a significant difference between the proportions of urban and suburban residents who favour construction of the nuclear plant? Use 1% level of significance. (10)
8. An analysis was performed to understand the effect of temperature on the electric power consumed by a nuclear plant in a city of Himachal Pradesh. Considering the other factors as constant, the data collected have been tabulated below:

Temperature (°F)	27	45	72	58	31	60	34	74
Electric power (wt)	250	285	320	295	265	298	267	321

- (i) Fit a simple linear regression model between temperature and electric power by finding out the estimates of intercept and slope.
- (ii) Predict the power consumption for a temperature of 85°F. (10)

SECTION - E (Compulsory)

9. (a) In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?

- (b) The resistance (x) of an electrical component has a probability density function, $f(x) = Ax(130 - x^2)$ for resistance values in the range $10 \leq x \leq 11$. Calculate the value of the constant A .
- (c) What is the probability mass function and expected (mean) value of Hyper geometric distribution?
- (d) Determine the mean and variance of a Beta distribution with parameters α and β .
- (e) Suppose that the random variable (X) has a Weibull distribution with shape parameter $a = 2.3$ and scale parameter $\lambda = 1.7$. Find $P(0.5 \leq X \leq 1.5)$.
- (f) Define unbiased and biased point estimators.
- (g) Describe Maximum likelihood estimator.
- (h) Define a type I error and type II error in testing a statistical hypothesis.
- (i) Which hypothesis test is used to check independence of attributes? Justify your answer.
- (j) Describe different types of lines of correlation. (10×2=20)