

Dec.-23-1208

MA-111 (Applied Mathematics-I) (Group-A&B)

B.Tech. 1st (CBCS/NEP)

(Common for all Branches)

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any five questions in all by selecting one each from section A, B, C and D. Section-E is compulsory.

SECTION - A

1. (i) Examine the convergence of the series:

$$\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \dots \infty. \quad (5)$$

- (ii) Discuss the convergence of the series:

$$1 - 2x + 3x^2 - 4x^3 + \dots \infty, \left(x < \frac{1}{2}\right). \quad (5)$$

2. (i) Test the following series for absolute convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}. \quad (5)$$

- (ii) Examine for term by term integration the series for which

$$f_n(x) = nx e^{-nx^2} \text{ in the intervals (i) } [0, 1] \text{ (ii) } [c, 1], 0 < c < 1. \quad (5)$$

SECTION - B

3. (i) Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ in $[-3, 0]$. (5)

- (ii) Use Cauchy's Mean Value theorem to evaluate

$$\lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{\log \frac{1}{x}}. \quad (5)$$

4. (i) Find the length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant. (5)

- (ii) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin \theta)$ and $y = a(1 + \cos \theta)$ about x-axis. (5)

SECTION - C

5. (i) In a plane triangle, find the maximum value of $\cos A \cos B \cos C$. (5)

- (ii) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}. \quad (5)$$

6. (i) If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial z}$. (5)

- (ii) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (5)

SECTION - D

7. (i) Evaluate $\iint_R e^{2x-3y} dx dy$ over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$. (5)

[P.T.O.]

(ii) Change the order of integration in the following integral and

evaluate: $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$. (5)

8. (i) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{xy dx}{\sqrt{x^2+y^2}}$ by changing to polar co-ordinates. (5)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (5)

SECTION - E
(Compulsory Question)

9. (i) Define Series with example.
 (ii) Define Cauchy integral test.
 (iii) State Lagrange's Cauchy mean value theorem.
 (iv) Define continuity of a function with example.
 (v) Define Taylor series.
 (vi) Define absolute convergence.
 (vii) Define double integral.
 (viii) Define alternating series.
 (x) Define definite integral.
 (x) Define change of order of integration in double integral.
 (10×2=20)