

Note: Attempt five questions in all, selecting one question each from Section A, B, C and D. Section E is compulsory.

SECTION-A

1. (a) Show that the vectors $x_1 = (1, 1, -1, 1)$, $x_2 = (1, -1, 2, -1)$ and $x_3 = (3, 1, 0, 1)$ are linearly dependent. Also find the relation between them. (5)

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} \quad (5)$$

2. (a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Hence find its inverse. (5)

(b) Find the values of k for which the system of equations $(3k - 8)x + 3y + 3z = 0$, $3x + (3k - 8)y + 3z = 0$ and $3x + 3y + (3k - 8)z = 0$ has a non-trivial solution. (5)

SECTION-B

3. (a) Define circular and hyperbolic functions. Also find the relation between circular and hyperbolic functions. (5)

(b) Separate the real and imaginary parts of $\tanh(x + iy)$. (5)

4. Prove that the function $f(z)$ defined by

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, (z \neq 0), f(0) = 0$$

is continuous and the Cauchy Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist. (10)

SECTION-C

5. (a) Examine the following function for extreme values

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2. \quad (5)$$

(b) Expand $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x - 1)$ and $(y - 1)$ up to third degree terms. (5)

6. (a) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (5)

(b) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate the same. (5)

SECTION-D

7. (a) Find grad ϕ , where $\phi = \log(x^2 + y^2 + z^2)$. (5)

(b) Find div F and curl F, where $F = x^2yz \, I + xy^2z \, J + xyz^2 \, K$. (5)

8. (a) Verify divergence theorem for $F = (x^2 - yz)I + (y^2 - zx)J + (z^2 - xy)K$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$. (5)

(b) Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the area enclosed by x-axis and upper half of the circle $x^2 + y^2 = a^2$. (5)

SECTION-E

9. (a) Define rank of matrix. Also give example.
- (b) Prove that eigen values of A and A' are same.
- (c) State Cayley Hamilton theorem.
- (d) Find Jacobian of $u = x^2 + y^2$, $v = x^2y^2$.
- (e) Find grad ϕ , where $\phi = \log(x^2 + y^2 + z^2)$.
- (f) Explain limit and continuity of complex functions.
- (g) Define an analytic function.
- (h) State Euler's theorem on Homogeneous functions.
- (i) Define gamma function. Give example.
- (j) State Stoke's theorem. (10×2=20)