[Total No. of Questions - 9] [Total No. of Printed Pages - 4]

Dec-24-0276 (CBCS)

MA-101 (Engineering Mathematics-I) (A&B)

B.Tech. 1st

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt Five questions in all, selecting one question each from section A, B, C and D. Q. No. 9 is compulsory.

SECTION - A

1. (a) Reduce the matrix to normal form and hence find the rank of the matrix.

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ (5)

(b) Show that the vectors $X_1 = (2, -1, 4)$, $X_2=(0, 1, 2)$, $X_3 = (6, -1, 16)$ are linearly dependent and find a relation between them. (5)

2. (a) Show that the equations $3x+4y+5z+=a$, $4x+5y+6z=b$, $5x+6y+7z=c$ do not have a solution unless $a+c=2b$. (5)

(b) Find the eigen values and eigen vectors of the matrices

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ (5)

SECTION - B

3. (a) If $i^{\alpha+i\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$. (5)

(b) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

(i) $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$ (ii) $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$ (5)

4. (a) Find the sum of the series

$\cos^2\theta - \frac{1}{3}\cos^3\theta\cos 3\theta + \frac{1}{5}\cos^5\theta\cos 5\theta \ldots\ldots\ldots\infty$ (5)

(b) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. (5)

SECTION - C

5. (a) If u = f(r) and x = rcosθ, y=rsinθ, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{f}f'(r)$. (5)

(b) If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)^{1/2}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u)$$ (5)

6. (a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to spherical polar co-ordinates. (5)

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y + z = 4 and z = 0. (5)

SECTION - D

7. (a) Prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$ (5)

   (b) Show that the surfaces $5x^2 - 2yz - 9x = 0$ and $4x^2y + z^3 - 4 = 0$ are orthogonal at the point (1, –1, 2). (5)

8. (a) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2, z = 0$. (5)

   (b) Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}dS$, where $\vec{F} = x^3\hat{i} + x^2y\hat{j} + x^2z\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = a^2$, $z = 0$, $z = b$. (5)

SECTION - E
(Compulsory question)

9. (a) Define rank of a matrix and write the procedure to find the rank of a matrix.

   (b) Prove that the eigen values of a matrix is the sum of the elements on the principal diagonal.

   (c) Split up into real and imaginary parts: $\coth(x + iy)$.

   (d) Prove that cos z is not bounded, where $z = x + iy$ is a complex variable.

   (e) If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$.

(f) If $u = \log \frac{x^4 + y^4}{x + y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

(g) Prove that $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$.

(h) Evaluate $\iint_R xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

(i) In what direction from the point (1, 1, –2), is the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ maximum? Also, find the maximum directional derivative.

(j) Prove that $\oint_C \vec{r}.d\vec{r} = 0$, where C is the simple closed curve.

(10×2=20)